

BFKL ansatz for BK equation in conformal basis

S.Bondarenko ^{a)} *, A.Prygarin ^{b),c)} †

^{a)} *University of Santiago de Compostela,
Santiago de Compostela, Spain*

^{b)} *HEP Department, School of Physics and Astronomy,
Raymond and Beverly Sackler Faculty of Exact Science,
Tel-Aviv University, Ramat Aviv, 69978, Israel*

^{c)} *II. Institut für Theoretische Physik,
Universität Hamburg ,
Luruper Chaussee 149, D-22761 Hamburg, Germany*

February 2, 2008

Abstract

The BK equation in the conformal basis is considered and analyzed. It is shown that at high energy a factorization of the coordinate and rapidity dependence should hold. This allows to simplify significantly the form of the equation under discussion. An analytical ansatz for the solution to the BK equation at high energies is proposed and analyzed. This analytical ansatz satisfies the initial condition at low energy and does not depend on both rapidity and the initial condition in the high energy limit. The case of the final rapidity being not too large is discussed and the properties of the transition region between small and large final rapidities have been studied.

1 Introduction

The scattering of two distinct object, such as hadron and nucleus, or DIS at high energy is described by so-called "fan" diagrams where only splitting of one pomeron into two is taken into account [1, 2, 3]. This situation was already discussed many years ago in the framework of the phenomenological pomeron [4], and some early attempts have been taken to generalize it for the case of QCD [5]. The equation for the "fan" diagrams in the operator expansion formalism was written by I.Balitsky [9], and in the dipole model framework [11] by Yu.Kovchegov [10]. The resulting Balitsky-Kovchegov (BK) equation is very well numerically studied in both, the saturation region and the region of small non-linearity (see

*Email: sergey@fpaxp1.usc.es

†E-mail: prygarin@post.tau.ac.il

[12, 13, 14] and [15, 16]). One of the important features of the BK equation is the presence of the saturation scale at high energies that increases exponentially with rapidity, and a geometrical scaling of the solution [17, 18, 19]. Despite the well understanding of the properties of the BK equation, the question of an exact analytical solution to the BK equation still remains open. This full analytical solution may be very useful in the applications of the BK equation to different scattering processes at high energies.

In the present paper we want to bridge the gap in the analytical study of the BK equation. We consider the formulation of the BK equation in the conformal basis and solve the problem using some simplifications. In the conformal BK equation we keep only the terms with zero conformal spin, which corresponds to leaving only the BFKL pomerons propagators in the "fan" diagrams. Due to highly complicated forms of the integrals appearing in the calculations the exact analytical solution is beyond the scope of the present paper. However, we do discuss the possible ways of finding this exact analytical solution. We propose an ansatz of the solution to the BK equation, which relates between high energy behavior and the given initial condition at zero rapidity for the pomeron field. This ansatz we call "phenomenological" or the BFKL ansatz for the solution to the BK equation. The reason for this is very simple, "phenomenological" because this solution is similar to phenomenological fan solution found in [4] (see also [20] for more details) and the BFKL since the found solution corresponds to the leading BFKL terms in the "fan" structure leading at high energies.

The said ansatz was obtained using the conformal invariance of the pomeron field and the action of the theory at high energy. The conformal invariance disappears when the final rapidity becomes not too large. This happens due to the influence of the non-invariant source that cannot be neglected at low energy. Therefore, for small final rapidities we consider a similar ansatz, which nevertheless does not possess property of the conformal invariance. Having two solutions for two different regions we are able to analyze the properties of the transition of the pomeron field from one region to another.

The present paper is organized as follows. In the next section we consider the BK equation in the usual coordinate formulation and rewrite it in the conformal basis. In Sec. 3 we consider the conformal invariance of the theory at high energy and obtain scaling properties of the pomeron field. In Sec. 4 we propose a possible ansatz of solution to the BK equation in the conformal basis at high energy and discuss the analytical structure of the solution. In Sec. 5 we consider an ansatz for the pomeron field at small values of rapidity. Sec. 6 is dedicated to the conditions and scales arising in the region where the transformation of the pomeron field from the region of low energy to the region of the high energy place. In the last section the conclusions and discussions are presented.

2 BK equation in conformal basis

We begin our consideration with the effective field theory of the interacting pomerons. It was shown by Braun (see Refs. [21, 22, 23, 24]) that one can describe the interacting pomerons in the large N_c limit in terms of the effective action. The effective action with pomeron splitting only reads

$$S = S_0 + S_I - S_E, \quad (1)$$

where S_0 is a free part of the action

$$S_0 = \int dy dy' d^2 \rho_1 d^2 \rho_2 d^2 \rho'_1 d^2 \rho'_2 \Phi^\dagger(y, \rho_1, \rho_2) G_{y-y'}^{-1}(\rho_1, \rho_2 | \rho'_1, \rho'_2) \Phi(y, \rho'_1, \rho'_2) \quad (2)$$

S_I is a interacting part of the action

$$S_I = \frac{2 \alpha_s^2 N_c}{\pi} \int dy \int \frac{d^2 \rho_1 d^2 \rho_2 d^2 \rho_3}{r_{12}^2 r_{23}^2 r_{31}^2} (L_{13} \Phi^\dagger(y, \rho_1, \rho_3)) \Phi(y, \rho_1, \rho_2) \Phi(y, \rho_2, \rho_3) \quad (3)$$

and S_E is a source term of the action

$$S_E = \int dy \int d^2 \rho_1 d^2 \rho_2 \Phi(y, \rho_1, \rho_2) \tau_A(y, \rho_1, \rho_2) + \Phi^\dagger(y, \rho_1, \rho_2) \tau_B(y, \rho_1, \rho_2) \quad (4)$$

The propagator of the theory is defined through the BFKL Hamiltonian [27, 28, 29] as follows

$$G_{y-y'}^{-1}(\rho_1, \rho_2 | \rho_1', \rho_2') = \left(\nabla_2^2 \nabla_1^2 \left(\frac{\partial}{\partial y} + H(\rho_1, \rho_2) \right) \right) \delta^2(\rho_1 - \rho_1') \delta^2(\rho_2 - \rho_2') \delta(y - y') \quad (5)$$

The source terms in Eq. (4) are the initial forms of the functions Φ and Φ^\dagger at rapidities $y = 0$ and $y = Y$ respectively

$$\Phi(y, \rho_1, \rho_2)_{y=0} = \tau_B(y, \rho_1, \rho_2) = \bar{\tau}_B(\rho_1, \rho_2) \delta(y) \quad (6)$$

$$\Phi^\dagger(y, \rho_1, \rho_2)_{y=Y} = \tau_A(y, \rho_1, \rho_2) = \bar{\tau}_A(\rho_1, \rho_2) \delta(y - Y) \quad (7)$$

The pomeron field $\Phi(y, r_i, r_j)$ in Eq. (2) and Eq. (3) is the generalized (skewed) gluon (parton) distribution written in coordinate representation, see [25, 26], and L_{13} is the Casimir operator of the conformal group

$$L_{13} = \rho_{13}^4 p_1^2 p_3^2 = \rho_{13}^4 \nabla_1^2 \nabla_3^2 \quad (8)$$

In this formalism the BK equation is the equation of motion for the pomeron field $\Phi(y, \rho_i, \rho_j)$

$$\left(\frac{\partial}{\partial y} + H(\rho_1, \rho_3) \right) \Phi(y, \rho_1, \rho_3) + \frac{2\alpha_s^2 N_c}{\pi} \int \frac{d^2 \rho_2 \rho_{31}^2}{\rho_{12}^2 \rho_{23}^2} \Phi(y, \rho_1, \rho_2) \Phi(y, \rho_2, \rho_3) = 0 \quad (9)$$

with the initial condition for the field given by Eq. (6).

For our further analysis it is more convenient to write Eq. (9) in the conformal basis of functions $E_\mu(\rho_1, \rho_2)$ (see Ref.[27])

$$E_\mu(\rho_1, \rho_2) = \left(\frac{\rho_{12}}{\rho_{10} \rho_{20}} \right)^h \left(\frac{\bar{\rho}_{12}}{\bar{\rho}_{10} \bar{\rho}_{20}} \right)^{\bar{h}} \quad (10)$$

where $\mu = \{h, \rho_0\}$, $h = \frac{1+n}{2} + i\nu$ and $\bar{h} = 1 - h^*$. For the sake of simplicity we adopted notation used in Ref.[23]. These functions are the eigenfunctions of L_{13} operator

$$L_{13} E_\mu(\rho_1, \rho_2) = \lambda_\mu^{-1} E_\mu(\rho_1, \rho_2) \quad (11)$$

with the eigenvalues

$$\lambda_\mu = \frac{1}{[(n+1)^2 + 4\nu^2][(n-1)^2 + 4\nu^2]} \quad (12)$$

The pomeron field can be expanded in this basis as follows (see [27])

$$\Phi(y, \rho_1, \rho_2) = \sum_\mu E_\mu(\rho_1, \rho_2) \Phi_\mu(y) \quad (13)$$

where

$$\Phi_\mu(y) = \int \frac{d^2 \rho_1 d^2 \rho_2}{\rho_{12}^4} E_\mu^*(\rho_1, \rho_2) \Phi(y, \rho_1, \rho_2) \quad (14)$$

In Eq. (13) and hereafter by the conformal summation one should understand

$$\sum_\mu = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu \frac{\nu^2 + \frac{n^2}{4}}{\pi^4} \int d^2 \rho_0 \quad (15)$$

Recasting the Lagrangian of the theory in the conformal basis (for more details see [26]) one obtains

$$S = \int dy \sum_{\mu} \left\{ \frac{1}{2} \Phi_{\mu}^{\dagger}(y) \lambda_{\mu}^{-1} \frac{\partial \Phi_{\mu}(y)}{\partial y} - \frac{1}{2} \Phi_{\mu}(y) \lambda_{\mu}^{-1} \frac{\partial \Phi_{\mu}^{\dagger}(y)}{\partial y} - \omega_{\mu} \lambda_{\mu}^{-1} \left(\Phi_{\mu}(y) \Phi_{\mu}^{\dagger}(y) - \frac{2\alpha_s^2 N_c}{\pi} \omega_{\mu}^{-1} V_{\tilde{\mu}, w, \nu} \Phi_{\mu}^{\dagger}(y) \Phi_w(y) \Phi_{\nu}(y) \right) + \Phi_{\mu}(y) \tau_{\mu_A} + \Phi_{\mu}^{\dagger}(y) \tau_{\mu_B} \right\} \quad (16)$$

with ω_{μ} being the BFKL eigenvalues

$$\omega_{\mu} = \omega_h = \bar{\alpha} \left(\psi(1) - \text{Re} \psi\left(\frac{|n| + 1}{2} + i\nu\right) \right) \quad (17)$$

and with the sources τ_{μ_A} and τ_{μ_B} for the fields Φ^{\dagger} and Φ at rapidities $y = Y$ and $y = 0$ correspondingly. The effective action in this form leads to the equation of motion for each Φ_{μ} field

$$\frac{\partial \Phi_{\mu}(y)}{\partial y} = \omega_{\mu} \Phi_{\mu}(y) - \frac{2\alpha_s^2 N_c}{\pi} \sum_{\mu_1, \mu_2} \Phi_{\mu_1}(y) \Phi_{\mu_2}(y) V_{\tilde{\mu}, \mu_1, \mu_2} \quad (18)$$

where $V_{\tilde{\mu}, \mu_1, \mu_2}$ is the triple pomeron vertex in the conformal basis given by

$$V_{\mu, \mu_1, \mu_2} = \int \frac{d^2 \rho_1 d^2 \rho_2 d^2 \rho_3}{\rho_{12}^2 \rho_{23}^2 \rho_{31}^2} E_{\mu}(\rho_1, \rho_2) E_{\mu_1}(\rho_2, \rho_3) E_{\mu_2}(\rho_3, \rho_1) \quad (19)$$

and $\tilde{\mu} = \{1 - h, \rho_0\}$. The expression of Eq. (18) is, essentially, the BK equation in the conformal basis and its solution is the main subject under consideration in the present paper.

Further simplifications of Eq. (19) using properties of the conformal group were performed in Ref.[31] and the simplified expression reads

$$V_{\mu_0, \mu_1, \mu_2} = \Omega(h_0, h_1, h_2) (z_0 - z_1)^{-\Delta_{01}} (z_1 - z_2)^{-\Delta_{12}} (z_2 - z_3)^{-\Delta_{23}} (\bar{z}_0 - \bar{z}_1)^{-\bar{\Delta}_{01}} (\bar{z}_1 - \bar{z}_2)^{-\bar{\Delta}_{12}} (\bar{z}_2 - \bar{z}_0)^{-\bar{\Delta}_{20}} \quad (20)$$

where $\Delta_{12} = h_1 + h_2 - h_0$ and $\bar{\Delta}_{12} = \bar{h}_1 + \bar{h}_2 - \bar{h}_0$. We follow the notation of Ref.[30, 31] and denote ρ_0 of μ_0 by z_0 , i.e. $\mu_0 = \{h_0, z_0\}$ in Eq. (20) and below. Using the simplified expression for the triple pomeron vertex in Eq. (20) we can write explicitly the nonlinear term in Eq. (18) as

$$\frac{1}{4} \frac{2\alpha_s^2 N_c}{\pi} \sum_{n_1, n_2 = -\infty}^{\infty} \int_{-\infty}^{\infty} d\nu_1 \frac{\nu_1^2 + \frac{n_1^2}{4}}{\pi^4} \int_{-\infty}^{\infty} d\nu_2 \frac{\nu_2^2 + \frac{n_2^2}{4}}{\pi^4} \int dz_1 \int dz_2 (z_0 - z_1)^{-\Delta_{01}} (z_1 - z_2)^{-\Delta_{12}} (z_2 - z_0)^{-\Delta_{20}} \quad (21)$$

$$\times \int d\bar{z}_1 \int d\bar{z}_2 (\bar{z}_0 - \bar{z}_1)^{-\bar{\Delta}_{01}} (\bar{z}_1 - \bar{z}_2)^{-\bar{\Delta}_{12}} (\bar{z}_2 - \bar{z}_0)^{-\bar{\Delta}_{20}} \Phi_{\mu_1}(y, z_1) \Phi_{\mu_2}(y, z_2) \Omega(1 - h_0, h_1, h_2)$$

where the Δ_{01} notation stands for h_0 to be replaced by $1 - h_0$ in the expression for Δ_{01} defined above. This comes from the fact that the expression for Eq. (19) given by Eq. (20) appears in Eq. (18) with one of the conformal vertex functions conjugated. To see the scaling properties of this nonlinear term we introduce dimensionless variables

$$w_1 = \frac{z_1}{z_0}, \quad w_2 = \frac{z_2}{z_0} \quad (22)$$

and their conjugate. In terms of these new dimensionless variables Eq. (21) reads

$$\frac{1}{4} \frac{2\alpha_s^2 N_c}{\pi} \sum_{n_1, n_2 = -\infty}^{\infty} \int_{-\infty}^{\infty} d\nu_1 \frac{\nu_1^2 + \frac{n_1^2}{4}}{\pi^4} \int_{-\infty}^{\infty} d\nu_2 \frac{\nu_2^2 + \frac{n_2^2}{4}}{\pi^4} z_0^{2 - \Delta_{01} - \Delta_{12} - \Delta_{20}} \bar{z}_0^{2 - \bar{\Delta}_{01} - \bar{\Delta}_{12} - \bar{\Delta}_{20}}$$

$$\begin{aligned}
& \times \int dw_1 \int dw_2 (1-w_1)^{-\Delta_{\bar{0}1}} (w_1-w_2)^{-\Delta_{12}} (w_2-1)^{-\Delta_{2\bar{0}}} \int d\bar{w}_1 \int d\bar{w}_2 (1-\bar{w}_1)^{-\bar{\Delta}_{\bar{0}1}} (\bar{w}_1-\bar{w}_2)^{-\bar{\Delta}_{12}} (\bar{w}_2-1)^{-\bar{\Delta}_{2\bar{0}}} \\
& \times \Phi_{\mu_1}(y, z_0 w_1, \bar{z}_0 \bar{w}_1) \Phi_{\mu_2}(y, z_0 w_2, \bar{z}_0 \bar{w}_2) \Omega(1-h_0, h_1, h_2)
\end{aligned} \tag{23}$$

As one can see from Eq. (23) the scaling properties of the triple pomeron term are merely determined from the properties of the pomeron field and the power of the dimensionful variable z_0 .

3 The scaling property of the pomeron field

In this section we show how the scaling properties of the pomeron field suggest an ansatz for solution of the conformal BK equation at high energies. We come back to the scaling independent value of the theory, to the action given by Eq. (16), and check the scaling properties of the pomeron field. We assume that our theory is conformal invariant at high energies. As a consequence of this fact the pomeron field scales accordingly to its dimensions

$$[\Phi_\mu(Y, z_0)] = z_0^{h-1} \bar{z}_0^{\bar{h}-1} \tag{24}$$

resulting in the following scaling property

$$\Phi_\mu(Y, \lambda z_0, \bar{\lambda} \bar{z}_0) = \lambda^{h-1} \bar{\lambda}^{\bar{h}-1} \Phi_\mu(Y, z_0, \bar{z}_0) \tag{25}$$

Next we analyze the scaling property of the source. We plug the equation of motion Eq. (18) into the action of the theory and obtain

$$S = \int dy \sum_\mu \Phi_\mu(y) \tau_{\mu_A}(y) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu \frac{\nu^2 + \frac{n^2}{4}}{\pi^4} \int d^2 z_0 \Phi_\mu(Y, z_0) \bar{\tau}_{\mu_A}(z_0), \tag{26}$$

where Y is the final rapidity of the process and $\bar{\tau}_{\mu_A}(z_0)$ is defined by

$$\tau_{\mu_A}(y) = \bar{\tau}_{\mu_A}(z_0) \delta(y - Y) \tag{27}$$

Using the scaling property of the pomeron field found in Eq. (25) one can derive the scaling of the source

$$\lambda^h \bar{\lambda}^{\bar{h}} \bar{\tau}_{\mu_A}(\lambda z_0, \bar{\lambda} \bar{z}_0) = \bar{\tau}_{\mu_A}(z_0, \bar{z}_0) \tag{28}$$

Another consequence of the scaling property of the pomeron field Eq. (25) is that it preserves unchanged the form of the equation of motion with the triple pomeron term given by Eq. (23). This observation and the condition of the invariance of the action under the rescaling prompts the following ansatz for the pomeron field at high energy

$$\Phi_\mu(y, z) = z^{h-1} \bar{z}^{\bar{h}-1} f^{(h, \bar{h})}(y) \tag{29}$$

It is easy to see that this ansatz satisfies all conditions which we have on the theory at high energy. Another important observation we made is the fact that Eq. (29) presents a manifestation of the holomorphic separability of the pomeron field at high energies.

Now we are in position to plug the ansatz Eq. (29) into Eq. (18) with the help of Eq. (21) and read out the relevant integrals over the coordinate variables. The coordinate integration factorizes and is performed in the Appendix A. The resulting equation is the integro-differential equation for $f^{(h, \bar{h})}(y)$ function given by

$$\frac{\partial f^{(h, \bar{h})}(y)}{\partial y} = \omega_h f^{(h, \bar{h})}(y) - \frac{\alpha_s^2 N_c}{2\pi} \sum_{h_1, h_2} f^{(h_1, \bar{h}_1)}(y) f^{(h_2, \bar{h}_2)}(y) \Omega(1-h, h_1, h_2) I(1-h, h_1, h_2) I(1-\bar{h}, \bar{h}_1, \bar{h}_2) \tag{30}$$

where the integrals $I(1-h, h_1, h_2)$ and $I(1-\bar{h}, \bar{h}_1, \bar{h}_2)$ are calculated in the Appendix A and the summation over h denotes

$$\sum_h = \sum_{n=-\infty}^{\infty} \int d\nu \frac{\nu^2 + \frac{n^2}{4}}{\pi^4} \quad (31)$$

The solution of Eq. (30) would give an exact analytical solution to the BK equation in the limit of high energy.

It is interesting to note that the form of the coordinate dependence of the function $\Phi_\mu(y)$ given by the ansatz Eq. (29) resembles the form of the momentum dependence of the solution of the BFKL equation for zero transferred momentum. The physical meaning of this dependence is different since the variables z and \bar{z} are conjugate to the transferred momentum while in the solution of the BFKL equation such power dependence arises in the transverse momentum of the pomeron. However, the question of a more subtle relation between the two cases is still to be answered. Another interesting observation, is that a factorized form of the ansatz Eq. (29) is a consequence of the fact that at high energy the solution of the equation of motion does not "remember" about the initial conditions and there is no another scale which can destroy this factorization.

4 Solution of BK equation in conformal basis at large final rapidities

As it was already mentioned Eq. (30) is our master one variable integro-differential equation, solution to which gives the full analytical solution to the BK equation at high energy. Unfortunately, being much simpler than the original BK equation it still quite complicated. The exact solution of Eq. (30) is beyond the scope of the present paper and we only wish to find the leading solution at high energies. We consider the case of zero conformal spin $n_i = 0$ which corresponds to the fan diagrams with only the BFKL pomeron propagators. For this case Eq. (30) reads

$$\begin{aligned} \frac{\partial f^\nu(y)}{\partial y} &= \omega_\nu f^\nu(y) - \frac{\alpha_s^2 N_c}{2\pi} \int_{-\infty}^{\infty} \frac{\nu_1^2}{\pi^4} d\nu_1 \int_{-\infty}^{\infty} \frac{\nu_2^2}{\pi^4} d\nu_2 f^{\nu_1}(y) f^{\nu_2}(y) \\ &\times \Omega\left(\frac{1}{2} - i\nu, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) I^2\left(\frac{1}{2} - i\nu, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) \end{aligned} \quad (32)$$

We want to investigate the properties of the solution to Eq. (32) assuming that the function $f^\nu(y)$ can be expanded in powers of ν

$$f^\nu(y) = \sum_{k=0}^{\infty} f_k(y) \nu^k \quad (33)$$

and plug into Eq. (32). To the zeroth order we obtain

$$\begin{aligned} \frac{\partial f_0(y)}{\partial y} &= \omega_0 f_0(y) - \frac{\alpha_s^2 N_c}{2\pi} f_0^2(y) \int_{-\infty}^{\infty} \frac{\nu_1^2}{\pi^4} d\nu_1 \int_{-\infty}^{\infty} \frac{\nu_2^2}{\pi^4} d\nu_2 \\ &\times \Omega\left(\frac{1}{2}, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) I\left(\frac{1}{2}, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) I\left(\frac{1}{2}, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) \end{aligned} \quad (34)$$

or the same equation in more compact form

$$\frac{\partial f_0(y)}{\partial y} = \omega_0 f_0(y) - \frac{\alpha_s^2 N_c}{2\pi} f_0^2(y) \mathbf{C} \quad (35)$$

with some coefficient

$$\mathbf{C} = \int_{-\infty}^{\infty} \frac{\nu_1^2}{\pi^4} d\nu_1 \int_{-\infty}^{\infty} \frac{\nu_2^2}{\pi^4} d\nu_2 \Omega\left(\frac{1}{2}, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) I^2\left(\frac{1}{2}, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) \quad (36)$$

The solution to Eq. (35) is readily found

$$f_0(y) = \frac{e^{\omega_0 y}}{Coef f + \frac{\alpha_s^2 N_c}{2\pi} \frac{C}{\omega_0} e^{\omega_0 y}} \quad (37)$$

where the unknown coefficient $Coef f$ is to be determined from the initial conditions for $f_0(y)$ function. Let's now simply write this condition in the following form

$$f_0(y=0) = F_{in} \quad (38)$$

which leads to

$$f_0(y) = \frac{\exp(\omega_0 y) F_{in}}{\frac{\alpha_s^2 N_c}{2\pi} F_{in} \frac{C}{\omega_0} (\exp(\omega_0 y) - 1) + 1}. \quad (39)$$

We see, that at asymptotically high energy this ansatz does not depend on the form of F_{in} , leaving the possibility for arbitrary form of F_{in} . We will precisely define this function latter. For obvious reasons we can call Eq. (39) the double leading ansatz to the solution of the BK equation. First, because we consider only the leading term dominant at high energies, which is the BFKL fan structure; second, the function $f_0(y)$ is the first term in the expansion in the powers of ν . For the reasons we can call Eq. (39) the BFKL ansatz for BK equation. This ansatz is also of a great interest since it is similar to the solution to the analog of the BK equation in the phenomenological pomeron theory, see for example [20].

As a next step in our analysis we partially reconstruct ν dependence of the function $f^\nu(y)$. The form of Eq. (39) suggests the following form of the solution with explicit ν dependence

$$f^\nu(y) = \frac{\exp(\omega_\mu y) F_{in}}{F_{in} g_0^{-1}(\nu) (\exp(\omega_\mu y) - 1) + 1}, \quad (40)$$

where $g_0(\nu)$ is the first term in the expansion of $f^\nu(y)$ in inverse powers of one pomeron exchange

$$f^\nu(y) = \sum_{k=0}^{\infty} g_k(\nu) e^{-k \omega_\nu y}. \quad (41)$$

The function $g_n(\nu)$ are found from the equation Eq. (32) in the form of

$$\begin{aligned} \omega_\nu(k+1) g_k(\nu) e^{-k \omega_\nu y} &= \sum_{k_1, k_2=0}^{\infty} \frac{\alpha_s^2 N_c}{2\pi} \int_{-\infty}^{\infty} \frac{\nu_1^2}{\pi^4} d\nu_1 \int_{-\infty}^{\infty} \frac{\nu_2^2}{\pi^4} d\nu_2 g_{k_1}(\nu_1) e^{-k_1 \omega_{\nu_1} y} g_{k_2}(\nu_2) e^{-k_2 \omega_{\nu_2} y} \\ &\times \Omega\left(\frac{1}{2} - i\nu, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) I^2\left(\frac{1}{2} - i\nu, \frac{1}{2} + i\nu_1, \frac{1}{2} + i\nu_2\right) \end{aligned} \quad (42)$$

As it was already mentioned due to complexity of ν dependent functions in the kernel of this equation we are only able to find the solution leading in energy. This means that we keep only terms with k, k_1 and k_2 equal to zero obtaining equation for $g_0(\nu)$ only

$$\omega_{\mu_1} g_0(\nu_1) = \frac{\alpha_s^2 N_c}{2\pi} \int_{-\infty}^{\infty} d\nu_2 \frac{\nu_2^2}{\pi^4} \int_{-\infty}^{\infty} d\nu_3 \frac{\nu_3^2}{\pi^4} g_0(\nu_2) g_0(\nu_3) \Omega I^2 \quad (43)$$

Plugging Eq. (40) into Eq. (29) we obtain, that when final rapidity of the process is large, the asymptotically leading solution of BK equation may be represented with the help of our ansatz

$$\Phi_\mu(y) = |z_0|^{-1+2i\nu} \frac{\exp(\omega_\mu y) F_{in}}{F_{in} g_0^{-1}(\nu) (\exp(\omega_\mu y) - 1) + 1} \quad (44)$$

As it must be, at the asymptotically large rapidity the pomeron field Eq. (44) can be written as

$$\Phi_\mu(y) = |z_0|^{-1+2i\nu} g_0(\nu), \quad (45)$$

that confirmed the scaling property of the pomeron field at large rapidities. In the case of the rapidity smaller then the final rapidity Y of the process, the factorization determined by Eq. (29) is already not necessary and may be broken. The possibility of the scenario when the final rapidity Y is small is discussed below.

The ansatz Eq. (40) correctly reproduce the property of BK equation at large rapidities, namely the independence of the ansatz on the initial conditions, the constant behavior at high energy and scaling independence of the action for the pomeron field. Nevertheless, the function Eq. (40) being used in the amplitude only at rapidity $y = Y$, could be defined at all values of rapidity. We find the form of F_{in} function from Eq. (44) taking $y = 0$ in the ansatz

$$\Phi_\mu(y)_{y=0} = |z_0|^{-1+2i\nu} F_{in} \quad (46)$$

As it was mentioned in the first section of the paper, the Lagrangian of the theory includes the source terms. This implies

$$\Phi_\mu(y) \delta(y) = \bar{\tau}_{\mu_B} \delta(y) \quad (47)$$

resulting in

$$|z_0|^{-1+2i\nu} F_{in} = \bar{\tau}_{\mu_B} \quad (48)$$

Plugging F_{in} from Eq. (48) back into Eq. (40) we obtain for our ansatz

$$\Phi_\mu(y) = \frac{\exp(\omega_\mu y) \bar{\tau}_{\mu_B}}{|z_0|^{1-2i\nu} \bar{\tau}_{\mu_B} g_0^{-1}(\nu) (\exp(\omega_\mu y) - 1) + 1} \quad (49)$$

The analytical ansatz of Eq. (49) interpolates between two desirable features of the behavior of the solution to BK equation. At high rapidity it is determined by the function which does not dependent on rapidity and initial condition of the problem, whereas at rapidity zero it is equal to the given initial function of the pomeron field. It is important to underline therefore, that the source field $\bar{\tau}_{\mu_B}$ in Eq. (49) is arbitrary due these properties of the ansatz. It is washed out at high rapidity and there are no special constraints on the functional form of this source. This source function may be arbitrary and may depend on some external scales of the problem.

5 Solution to BK equation in conformal basis at small final rapidities

In this section we consider a solution at small final rapidity Y . In this case the asymptotic expansion in powers of $e^{-\omega_\mu y}$ in Eq. (44) is not valid anymore. The proposed ansatz given in Eq. (49) does not describe correctly the solution to the BK equation and does not provide the scaling invariant solution for the amplitude as well. It is not surprising since we do not expect that at low final rapidity the solution will preserve this scaling invariance property. Indeed, let us return to the triple pomeron term in the equation of motion Eq. (23), keeping only zero conformal spins in the formulae. Clearly, all our previous consideration are valid if we can justify the asymptotic expansion of the pomeron field in the form

$$\Phi_\mu(y) = \sum_{k=0}^{\infty} g_k^\mu(z) e^{-k\omega_\mu y} \quad (50)$$

However, if Y is small another expansion should hold

$$\Phi_\mu(y) = \sum_{k=1}^{\infty} \bar{g}_k^\mu(z) e^{k\omega_\mu y}. \quad (51)$$

Similar the previous case we put the expansion of Eq. (51) into the equation of motion keeping only the first term of this expansion. It is easy to see that at small rapidities the triple pomeron term is not enhanced by rapidity exponential and can be safely neglected. In this case we obtain the BFKL pomeron solution which corresponds to the first term in the expansion Eq. (51)

$$\Phi_\mu(y) = \bar{\tau}_{\mu_B} e^{\omega_\mu y} \quad (52)$$

and determines the form of the action

$$S = \int_{-\infty}^{\infty} d\nu \frac{\nu^2}{\pi^4} \int d^2 z_0 \bar{\tau}_{\mu_B}(z_0) e^{\omega_\mu y} \bar{\tau}_{\mu_A}(z_0) \quad (53)$$

Similar to the case of the pomeron field we deduce from the form of the action the scaling property of the $\bar{\tau}_{\mu_B}$, namely,

$$\bar{\tau}_{\mu_B}(\lambda z_0, \bar{\lambda} \bar{z}_0) = \lambda^{-1+2i\nu} \bar{\tau}_{\mu_B}(z_0, \bar{z}_0) \quad (54)$$

Naturally, for an arbitrary source field $\bar{\tau}_{\mu_B}$ such a strong constraint, in general, is not satisfied. Therefore, we do not expect the invariance of the action under the rescaling of the variable z_0 at small energies. Such an invariance restores at large energies, when the dependence of the amplitude on the source $\bar{\tau}_{\mu_B}$ disappears.

As a next step in our discussion we consider the first two terms in the expansion Eq. (51) of the pomeron field at small energy

$$\Phi_\mu(y) = (\bar{\tau}_\mu(z_0) - \bar{g}_1^\mu(z_0)) e^{\omega_\mu y} + \bar{g}_1^\mu(z_0) e^{2\omega_\mu y}. \quad (55)$$

Using equation of motion we obtain the equation for the $\bar{g}_1^\mu(z_0)$ function from this expansion

$$\bar{g}_1^\mu(z_0) e^{2\omega_\mu y} \omega_\mu = -\frac{2\alpha_s^2 N_c}{\pi} \sum_{\mu_1, \mu_2} V_{\bar{\mu}, \mu_1, \mu_2} (\bar{\tau}_{\mu_{1B}}(z_1) - \bar{g}_1^{\mu_1}(z_1)) (\bar{\tau}_{\mu_{2B}}(z_2) - \bar{g}_1^{\mu_2}(z_2)) e^{(\omega_{\mu_1} + \omega_{\mu_2})y} \quad (56)$$

In this way we can consider an expansion with any arbitrary number of terms obtaining a chain of the equations similar to Eq. (56) with involved interference terms between the functions $\bar{g}_i^\mu(z_0)$. Instead, we find an ansatz that have a structure of the expansion Eq. (51) and will coincide with the expansion Eq. (51) to some order. As the simplest example we take expansion up to the second order and the form of this ansatz is borrowed from Eq. (49) as follows

$$\Phi_\mu(y) = \frac{\exp(\omega_\mu y) \bar{\tau}_{\mu_B}}{|z_0|^{1-2i\nu} \bar{\tau}_{\mu_B} \bar{g}_0(\nu, z_0) (\exp(\omega_\mu y) - 1) + 1} \quad (57)$$

Expanding Eq. (57) and comparing the first two terms of this expansion to the Eq. (55) we obtain

$$\bar{g}_1^\mu(z_0) = -|z_0|^{1-2i\nu} \bar{\tau}_{\mu_B}^2 \bar{g}_0(\nu, z_0) \quad (58)$$

We use this expression in Eq. (56) to obtain the equation for the function $\bar{g}_0(\nu, z_0)$, namely,

$$|z_0|^{1-2i\nu} \bar{\tau}_{\mu_B}^2 \bar{g}_0(\nu, z_0) e^{2\omega_\mu y} \omega_\mu =$$

$$\frac{2\alpha_s^2 N_c}{\pi} \sum_{\mu_1, \mu_2} V_{\bar{\mu}, \mu_1, \mu_2} \bar{\tau}_{\mu_{1B}}(z_1) \bar{\tau}_{\mu_{2B}}(z_2) (1 - |z_1|^{1-2i\nu_1} \bar{\tau}_{\mu_{1B}} \bar{g}_0(\nu_1, z_1)) (1 - |z_2|^{1-2i\nu_2} \bar{\tau}_{\mu_{2B}} \bar{g}_0(\nu_2, z_2)) e^{(\omega_{\mu_1} + \omega_{\mu_2})y}. \quad (59)$$

Further simplification of the obtained equations is possible if one assumes that

$$1 < |z|^{1-2i\nu} \bar{\tau}_{\mu_B} \bar{g}_0(\nu, z). \quad (60)$$

in the expansion Eq. (51). This condition means a smallness of the source multiplied by the triple pomeron vertex function. Indeed, there is a suppression of this term compared to unity due to α_s^2 in front of Eq. (59) if the source does not contain some large number (number of nucleons in nucleus, for example). In this case expanding the pomeron field Eq. (57) one obtains

$$\Phi_\mu(y) = \exp(\omega_\mu y) \bar{\tau}_{\mu_B} - |z_0|^{1-2i\nu} \bar{\tau}_{\mu_B}^2 \bar{g}_0(\nu, z_0, y) e^{2\omega_\mu y} \quad (61)$$

and the equation Eq. (59) becomes

$$|z_0|^{1-2i\nu} \bar{\tau}_{\mu_B}^2 \bar{g}_0(\nu, z_0, y) e^{2\omega_\mu y} \omega_\mu = \frac{2\alpha_s^2 N_c}{\pi} \sum_{\mu_1, \mu_2} V_{\mu, \mu_1, \mu_2} \bar{\tau}_{\mu_{1B}}(z_1) \bar{\tau}_{\mu_{2B}}(z_2) e^{(\omega_{\mu_1} + \omega_{\mu_2})y} \quad (62)$$

In Eq. (62) we introduced a rapidity dependence in the function \bar{g}_0 as the price for this simplification

$$\bar{g}_0(\nu, z_0) \rightarrow \bar{g}_0(\nu, z_0, y) \quad (63)$$

since in Eq. (62) this function possess some subdominant rapidity corrections. Thus in the case when Eq. (62) may be used instead Eq. (59) our ansatz becomes

$$\Phi_\mu(y) = \frac{\exp(\omega_\mu y) \bar{\tau}_{\mu_B}}{|z_0|^{1-2i\nu} \bar{\tau}_{\mu_B} \bar{g}_0(\nu, z_0, y) (\exp(\omega_\mu y) - 1) + 1}. \quad (64)$$

The obtained equations are more complicated then that of Eq. (44) in the high rapidity limit, because there is no factorization of z and ν variables.

6 The pomeron field at all rapidities and a transition region between small and large values of rapidity

The obtained ansatzs for large and small values of final rapidity indicate, that the only function that will be changed due to the rapidity evolution will be a g function in the denominators of the ansatz. Therefore, as a definition of the large and small rapidities we could use a following observations. Between these two rapidity regions of the solution, at asymptotically large rapidity and small rapidity, exists a region of the transition between growing and saturated behavior of the pomeron field. In general, this region may be defined as a region where we can instead expansion in exponents $e^{\omega_\mu y}$ we use a asymptotic expansion in the exponents $e^{-\omega_\mu y}$ and vice versa. Therefore, from the forms of Eq. (49) and Eq. (57) in this region may be defined by the following conditions

$$|z_{cr}|^{1-2i\nu} \bar{\tau}_{\mu_B}(z_{cr}) g_0^{-1}(\nu) (\exp(\omega_\mu y_{cr}) - 1) \propto 1, \quad (65)$$

and

$$|z_{cr}|^{1-2i\nu} \bar{\tau}_{\mu_B}(z_{cr}) \bar{g}_0(\nu, z_{cr}) (\exp(\omega_\mu y_{cr}) - 1) \propto 1 \quad (66)$$

This, in analogy with the usual definition of saturation momenta, will define the critical scale of the z_0 variable in the conformal basis

$$|z_{cr}|^{1-2i\nu} \bar{\tau}_{\mu_B}(z_{cr}) \propto g_0(\nu) (\exp(\omega_\mu y_{cr}) - 1)^{-1}. \quad (67)$$

for which the transition occurs. It is interesting to note, that the source $\bar{\tau}_{\mu_B}(z_{cr})$ in Eq. (67) brings some external scales dependence in Eq. (67), which defines the character of transition. We also could

consider the constraint Eq. (67) as a definition of some "transition" rapidity y_{cr} for fixed values of the final rapidity Y , vector z_0 and arbitrary external scales inside the source $\bar{\tau}_{\mu_B}$

$$y_{cr} \propto \frac{1}{\omega_\mu} \ln \left(1 + g_0(\nu) |z_{cr}|^{-1+2i\nu} \bar{\tau}_{\mu_B}^{-1}(z_{cr}) \right). \quad (68)$$

Comparing Eq. (65) and Eq. (66) we see that one can define the transition region as a region, where the following relation is satisfied

$$g_0^{-1}(\nu) \approx \bar{g}_0(\nu, z_0)_{z_0=z_{cr}} \quad (69)$$

This relation shows, inter alia, that if we find the functional form of $\bar{g}_0(\nu, z_0)$, then this function must satisfy the convergent expansion in z_0 around some value z_{cr} determined by Eq. (69)

$$\bar{g}_0(\nu, z_0) = \sum_{n=0}^{\infty} g_n^{-1}(\nu) (z_0 - z_{cr})^n. \quad (70)$$

So, with the use of the $\bar{g}_0(\nu, z_0)$ function we could find the $g_0^{-1}(\nu)$ function through Eq. (70) as well. Of course, for that we need to know a form of the $\bar{g}_0(\nu, z_0)$ function, that for general case is not easy. Therefore, instead, we could define as a transition region the region where both terms of expansion Eq. (61) are equal

$$\exp(\omega_\mu y) \bar{\tau}_{\mu_B} = |z_0|^{1-2i\nu} \bar{\tau}_{\mu_B}^2 \bar{g}_0(\nu, z_0, y) e^{2\omega_\mu y}. \quad (71)$$

Using the Eq. (62) we finally will obtain condition for the region where two solutions are overlapping

$$e^{\omega_\mu y_{cr}} \bar{\tau}_{\mu_B}(z_{cr}) = \frac{2\alpha_s^2 N_c}{\pi} \sum_{\mu_1, \mu_2} V_{\bar{\mu}, \mu_1, \mu_2} \bar{\tau}_{\mu_{1B}}(z_1) \bar{\tau}_{\mu_{2B}}(z_2) e^{(\omega_{\mu_1} + \omega_{\mu_2}) y_{cr}}. \quad (72)$$

Clearly, this is a screening condition on the sources of the problem, which defines values of z_{cr} and y_{cr} for which the source of the projectile will be screened from the target by the triple pomeron interactions.

7 The accuracy of ansatz

The source of possible corrections to the pomeron field at large rapidities are the coefficients $g_k(\nu)$ for different k in the series expression for the pomeron field

$$\Phi_\mu(y, z) = z^{h-1} \bar{z}^{\bar{h}-1} \sum_{k=0}^{\infty} g_k(\nu) e^{-k\omega_\nu y} \quad (73)$$

In the similar expansion for the phenomenological pomeron the ration of the coefficients of the successive terms is proportional to α_s . Based on this information we make an assumption that this is also the case in the expansion Eq. (73) and we only need to find the overall normalization, i.e. the order in α_s of the first term in the expansion.

We plug this series into the equation of motion and write the first two equation from the chain of equations Eq. (42). In integrals over ν we assume a contribution from such regions of ν in which the BFKL structure of the series Eq. (73) is kept, namely, from the "diffusion regions" of ν with $e^{\omega_\nu y} \approx e^{\omega_0 y}$. In the first order of expansion in $e^{-k\omega_\nu y}$ we obtain Eq. (43)

$$\omega_{\mu_1} g_0(\nu_1) = \frac{\alpha_s^2 N_c}{2\pi} \int_{-\infty}^{\infty} d\nu_2 \frac{\nu_2^2}{\pi^4} \int_{-\infty}^{\infty} d\nu_3 \frac{\nu_3^2}{\pi^4} g_0(\nu_2) g_0(\nu_3) \Omega I^2 \quad (74)$$

One can easily see from counting the powers of α_s that the coefficient function $g_0(\nu)$ is of the order of $1/\alpha_s$.

To show what are the limitations for the accuracy of the proposed ansatz we expand the expression in Eq. (49) for the large values of rapidity

$$\Phi_\mu(y, z) = z^{h-1} \bar{z}^{\bar{h}-1} g_0(\nu) \left(1 + e^{-\omega_\mu y} \left(1 - \frac{z^{h-1} \bar{z}^{\bar{h}-1}}{\bar{\tau}_\mu} g_0(\nu) \right) + e^{-2\omega_\mu y} \left(1 - \frac{z^{h-1} \bar{z}^{\bar{h}-1}}{\bar{\tau}_\mu} g_0(\nu) \right)^2 + \dots \right) \quad (75)$$

or, approximately,

$$\Phi_\mu(y, z) = z^{h-1} \bar{z}^{\bar{h}-1} g_0(\nu) \left(1 - e^{-\omega_\mu y} \frac{z^{h-1} \bar{z}^{\bar{h}-1}}{\bar{\tau}_\mu} g_0(\nu) + e^{-2\omega_\mu y} \left(\frac{z^{h-1} \bar{z}^{\bar{h}-1}}{\bar{\tau}_\mu} g_0(\nu) \right)^2 + \dots \right) \quad (76)$$

provided we neglect subleading terms in α_s assuming $1 < 1/\alpha_s$. We see that the expansion Eq. (76) properly reproduces the expected $(\alpha_s^k e^{(k-1)\omega y})^{-1}$ behavior for each term, in the agreement with that of the phenomenological pomeron.

Using this α_s structure of the expansion Eq. (76) one can easily see that the series converges for rapidities in the region of

$$y > \frac{1}{\alpha_s} \ln\left(\frac{1}{\alpha_s}\right) \quad (77)$$

Based on our discussion we conclude that the proposed ansatz Eq. (64) has both the correct high energy behavior and the leading α_s expansion in the rapidity region given by Eq. (77). If one wishes to take into account higher order corrections in α_s in the expansion Eq. (73), the proper way to do this is to introduce a new form of the ansatz

$$\Phi_\mu(y) = \frac{\exp(\omega_\mu y) \bar{\tau}_{\mu_B}}{|z_0|^{1-2i\nu} \bar{\tau}_{\mu_B} F(\exp(\omega_\mu y) - 1) + 1} \quad (78)$$

with some function

$$F = \sum_{n=-1}^{\infty} \alpha_s^n f_n(\nu, y) \quad (79)$$

where the all higher corrections are encoded.

It is a straightforward procedure to show in a similar way that the same arguments also hold for the low rapidity expansion where one expands the ansatz in powers of $e^{\omega_\mu y}$. In this case the validity of the expansion is restricted to the rapidity region given by

$$y < \frac{1}{\alpha_s} \ln\left(\frac{1}{\alpha_s}\right) \quad (80)$$

In order to account for higher order corrections in α_s in the ansatz Eq. (57) one can also make a use of some function F in a way similar to that of Eq. (78). However, in this case the function F possesses a different from Eq. (79) form of expansion in the powers of α_s

$$F = \sum_{n=1}^{\infty} \alpha_s^n f_n(\nu, y) \quad (81)$$

8 Conclusion

In the present paper we discussed possible ways of an analytical solution to the BK equation in the conformal basis. We suggested the following ansatz for the solution of BK equation

$$\Phi_\mu(y) = \frac{\exp(\omega_\mu y) \bar{\tau}_{\mu_B}}{|z_0|^{1-2i\nu} \bar{\tau}_{\mu_B} F(\exp(\omega_\mu y) - 1) + 1} \quad (82)$$

where the form of the function F in the denominator of Eq. (82) depends on the region of rapidity where the solution is considered. The problem, therefore, is reduced to the evolution of this unknown function F with rapidity. Assuming the conformal invariance of the theory at high energy we simplify our problem proposing factorization of the coordinate dependence of the pomeron field in the this limit. This makes it possible to separate the coordinate and rapidity dependence resulting into Eq. (30). This equation is still not easy to solve, but it allows us to investigate the energy dependence of the solution to the BK equation not mixing it with the coordinate degrees of freedom. The next important simplification we make is keeping only the BFKL structure of the "fan" diagrams thus reducing Eq. (30) to Eq. (32). This stems from the well known fact that the BFKL propagators present the leading contribution at high energy in such diagrams.

We find a solution to Eq. (32) which correctly describes high energy behavior of the exact solution and at the same time satisfies the initial condition at zero rapidity. Its expression is given by Eq. (49) and has the same energy structure as the phenomenological "fan" amplitude. The matching between the correct high energy behavior and the fulfillment of the initial condition has also another aspect. It is related to the fact that a strong condition on the source given by Eq. (54) is not satisfied in general, and thus dependence on a source breaks the conformal invariance of the solution. As one can easily see from the form of the solution Eq. (49) at high energy the dependence on the source disappears restoring the conformal invariance.

As a next step in our discussion we consider a question of a transition region in rapidity where a high energy solution ansatz transforms into the low energy one. This region can be thought of as one where the conformal invariance of the theory is restored or where the small rapidity expansion can be replaced by the asymptotic expansion. In this case some critical conformal scale can be introduced through Eq. (65)-Eq. (67). Another way to find the behavior and the parametrical form of the critical scale is to match between of low and high energy ansatzs of the solution given by Eq. (57) and Eq. (49), respectively. In this case the critical scale z_{cr} may be defined as a scale where the function $\bar{g}_0(\nu, z)$ coincides with the function g_0^{-1} .

It should be mentioned that the proposed ansatz is only an approximation to the full solutions given by series Eq. (41) and Eq. (51). Instead of using ansatz Eq. (49) and Eq. (57) one can develop perturbative calculations and obtain a chain of equations similar to ones given by Eq. (32) for large rapidities and by Eq. (56) for of small rapidities. The calculation of coefficient functions of the expansion Eq. (51) valid for small rapidities corresponds to the calculation of the diagrams in the perturbative expansion. Therefore, the first equation (Eq. (56) or Eq. (59)) from a chain of equations for the coefficient functions is similar to ones obtained in Ref. [32] for the simplest "fan" diagrams. The only difference between our result and that of Ref. [32] is the conformal basis and normal coordinate representations correspondingly. The physical interpretation of the expansion Eq. (41) is not so clear. The expansion in the negative powers of exponents of rapidity cannot be put into one to one correspondence with diagrams. The high energy behavior of the solution to the BK equation in the coordinate representation is well known (see [33]) and the question of the relation between the conformal and coordinate representation at large rapidities will be addressed in our further studies.

In this paper we considered the expansion of the coordinate dependent pomeron fields only in the conformal basis. Of course, for the practical applications the coordinate representation of the pomeron

field is more useful and convenient, but the expansion in conformal basis presents a more suitable framework for the investigation of the energy dependence properties of the pomeron field. These properties will determine the energy dependence of the full solution. The task of the inverse transformation of the found ansatz into the coordinate basis we leave , as we mentioned before, for further publications.

Acknowledgments

Authors would like to thank L.Lipatov for the helpful advises on the subject of the paper. A.P is grateful to the Santiago de Compostela University and personally to N.Arnesto and C.Pajares for their hospitality during the stay in Santiago de Compostela. A.P. would like to express his deep appreciation to J.Bartels for his hospitality at the University of Hamburg where the present work was completed. This research was supported in part by the Israel Science Foundation, founded by the Israeli Academy of Science and Humanities, by a grant from Ministry of Science, Culture and Sport, Israel and the Russian Foundation for Basic research of the Russian Federation. This paper was supported by the Ministerio de Educacion y Ciencia of Spain under project FPA2005-01963, and by Xunta de Galicia (Conselleria de Educacion).

Appendix A:

In this Appendix we calculate the integral which appears in Eq. (23). To do this we want to rewrite it in terms of some dimensionless variables and use integral representation of the hypergeometric functions. First we rescale the variables z_1 and z_2 as follows

$$\begin{aligned} \int_{-\infty}^{\infty} z_1^{\gamma_1} z_2^{\gamma_2} (z_0 - z_1)^{-\Delta_{\tilde{0}1}} (z_1 - z_2)^{-\Delta_{12}} (z_2 - z_0)^{-\Delta_{2\tilde{0}}} dz_1 dz_2 &= (-1)^{-\Delta_{12}-\Delta_{2\tilde{0}}} \bar{z}_0^{\gamma_1+\gamma_2+2-\Delta_{\tilde{0}1}-\Delta_{12}-\Delta_{2\tilde{0}}} \\ &\times \int_{-\infty}^{\infty} \left(\frac{z_1}{z_0}\right)^{\gamma_1} \left(\frac{z_2}{z_0}\right)^{\gamma_2-\Delta_{12}} \left(1 - \frac{z_1}{z_0}\right)^{-\Delta_{\tilde{0}1}} \left(1 - \frac{z_1}{z_0} \frac{z_2}{z_0}\right)^{-\Delta_{12}} \left(1 - \frac{z_2}{z_0}\right)^{-\Delta_{2\tilde{0}}} d\left(\frac{z_1}{z_0}\right) d\left(\frac{z_2}{z_0}\right) = \end{aligned} \quad (\text{A.1})$$

$$(-1)^{-\Delta_{12}-\Delta_{2\tilde{0}}} \bar{z}_0^{\gamma_1+\gamma_2+2-\Delta_{\tilde{0}1}-\Delta_{12}-\Delta_{2\tilde{0}}} \int_{-\infty}^{\infty} w_1^{\gamma_1} w_2^{\gamma_2-\Delta_{12}} (1 - w_1)^{-\Delta_{\tilde{0}1}} \left(1 - \frac{w_1}{w_2}\right)^{-\Delta_{12}} (1 - w_2)^{-\Delta_{2\tilde{0}}} dw_1 dw_2$$

where $w_1 = z_1/z_0$ and $w_2 = z_2/z_0$. Now we perform the integration over w_1 . The relevant integral is given by

$$I_1 \equiv \int_{-\infty}^{\infty} w_1^{\gamma_1} (1 - w_1)^{-\Delta_{\tilde{0}1}} \left(1 - \frac{w_1}{w_2}\right)^{-\Delta_{12}} dw_1 \quad (\text{A.2})$$

As one can see this integral reminds the integral representation of the Gauss hypergeometric function

$$\int_0^1 t^{b-1} (1-t)^{-b+c-1} (1-tz)^{-a} dt = \frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)} {}_2F_1(a, b, c, z) \quad (\text{A.3})$$

but the care about the limits should be taken. We split the limits of the integration in Eq. (A.3) as follows

$$\int_{-\infty}^{\infty} = \int_{-\infty}^0 + \int_0^1 + \int_1^{\infty} \quad (\text{A.4})$$

The second term on the rhc in Eq. (A.4) is just the integral representation of the hypergeometric function given in Eq. (A.3). The third term can be brought to the form of by substitution $w = 1/t$ and in this case one obtains the following identity

$$\int_1^{\infty} t^{b-1} (1-t)^{-b+c-1} (1-tz)^{-a} dt = (-)^{-a-b+c-1} \frac{\Gamma(a-c+1)\Gamma(c-b)}{\Gamma(a-b+1)} {}_2F_1\left(a, a-c+1, a-b+1, \frac{1}{z}\right) \quad (\text{A.5})$$

Thus the integral $(0, \infty)$ is obtained by summing Eq. (A.3) and Eq. (A.5). The remaining part $(-\infty, 0)$ is obtained from the integral $(0, \infty)$ by substituting $w = 1 - 1/t$ and reads

$$\int_{-\infty}^0 t^{b-1} (1-t)^{-b+c-1} (1-tz)^{-a} dt = (-)^{-c} \frac{\Gamma(b)\Gamma(a-c)}{\Gamma(a-c+b+1)} {}_2F_1\left(a, c-b, a-b+1, \frac{1}{1-z}\right) \quad (\text{A.6})$$

Summing the expressions of Eq. (A.3), Eq. (A.5) and Eq. (A.6) we can write the full expression to be used for w_1 integration as follows.

$$\int_{-\infty}^{\infty} t^{b-1} (1-t)^{-b+c-1} (1-tz)^{-a} dt = (-)^{-c} \frac{\Gamma(b)\Gamma(a-c)}{\Gamma(a-c+b+1)} {}_2F_1\left(a, c-b, a-b+1, \frac{1}{1-z}\right) + \quad (\text{A.7})$$

$$+ (-)^{-a-b+c-1} \frac{\Gamma(a-c+1)\Gamma(c-b)}{\Gamma(a-b+1)} {}_2F_1\left(a, a-c+1, a-b+1, \frac{1}{z}\right) + \frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)} {}_2F_1(a, b, c, z)$$

Comparing Eq. (A.2) and Eq. (A.7) we readily identify the parameters of Eq. (A.7) as

$$a = \Delta_{12}, \quad b = 1 + \gamma_1, \quad c = \gamma_1 - \Delta_{\bar{0}1} + 2, \quad z = \frac{1}{w_2} \quad (\text{A.8})$$

Because of the inverse dependence of w_2 on z in Eq. (A.8) it is more convenient for the further integration over w_2 to rewrite Eq. (A.7) in terms of the hypergeometric functions of the same argument $\frac{1}{z}$. This can be done using useful identities for the hypergeometric functions as follows (see **15.3.4** and **15.3.7** in [34])

$${}_2F_1(a, b, c; \frac{1}{1-z}) = (-1)^a (1-z)^a z^{-a} {}_2F_1(a, c-b, c; \frac{1}{z}) \quad (\text{A.9})$$

and

$$\begin{aligned} {}_2F_1(a, b, c; z) &= (-1)^a z^{-a} \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)} {}_2F_1(a, 1-c+a, 1-b+a; \frac{1}{z}) \\ &+ (-1)^b z^{-b} \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)} {}_2F_1(b, 1-c+b, 1-a+b; \frac{1}{z}) \end{aligned} \quad (\text{A.10})$$

With the help of Eq. (A.9) and Eq. (A.10) the integral in Eq. (A.7) reads

$$\int_{-\infty}^{\infty} t^{b-1} (1-t)^{-b+c-1} (1-tz)^{-a} dt = C_1 \cdot {}_2F_1\left(b, b-c+1, -a+b+1, \frac{1}{z}\right) + C_2 \cdot {}_2F_1\left(a, a-c+1, a-b+1, \frac{1}{z}\right)$$

with the functions C_1 and C_2 given by

$$C_1 = (-1)^b \left(\frac{1}{z}\right)^b \frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)}$$

and

$$C_2 = (-1)^{-a-b+c-1} \frac{\Gamma(a-c+1)\Gamma(c-b)}{\Gamma(a-b+1)} + (-1)^a \left(\frac{1}{z}\right)^a \frac{\Gamma(c-b)\Gamma(b-a)}{\Gamma(c-a)} + (-1)^c \left(1 - \frac{1}{z}\right)^a \frac{\Gamma(b)\Gamma(a-c)}{\Gamma(a+b-c+1)}$$

Thus with the help of Eq. (A.8) we identify the required integral Eq. (A.2) as

$$I_1 = C_1 \cdot {}_2F_1(1 + \gamma_1, \Delta_{\bar{0}1}, 2 + \Delta_{12} + \gamma_1, w_2) + C_2 \cdot {}_2F_1(\Delta_{12}, \Delta_{12} - \gamma_1 + \Delta_{\bar{0},1} - 1, \Delta_{12} - \gamma_1, w_2) \quad (\text{A.11})$$

with the functions C_1 and C_2 given by

$$C_1 = (-1)^{1+\gamma_1} w_2^{1+\gamma_1} \frac{\Gamma(1 + \gamma_1)\Gamma(1 - \Delta_{\bar{0}1})}{\Gamma(\gamma_1 - \Delta_{\bar{0}1} + 2)}$$

and

$$\begin{aligned}
C_2 = & (-1)^{\Delta_{\tilde{0}1} + \Delta_{12}} \frac{\Gamma(\Delta_{\tilde{0}1} + \Delta_{12} - \gamma_1 - 1) \Gamma(1 - \Delta_{\tilde{0}1})}{\Gamma(\Delta_{12} - \gamma_1)} + (-1)^{\Delta_{12}} w_2^{\Delta_{12}} \frac{\Gamma(1 - \Delta_{\tilde{0}1}) \Gamma(1 + \gamma_1 - \Delta_{12})}{\Gamma(2 + \gamma_2 - \Delta_{\tilde{0}1} - \Delta_{12})} \\
& + (-1)^{\gamma_1 - \Delta_{\tilde{0}1} + 2} (1 - w_2)^{\Delta_{12}} \frac{\Gamma(1 + \gamma_1) \Gamma(\Delta_{\tilde{0}1} + \Delta_{12} - \gamma_1 - 2)}{\Gamma(\Delta_{\tilde{0}1} + \Delta_{12})} \quad (\text{A.12})
\end{aligned}$$

The next step is to perform integration over variable w_2 . From Eq. (A.1) with the definition of Eq. (A.2) we see that the integral over w_2 reads

$$\int_{-\infty}^{\infty} I_1 w_2^{\gamma_2 - \Delta_{12}} (1 - w_2)^{-\Delta_{2\tilde{0}}} dw_2 \quad (\text{A.13})$$

It is clear from Eq. (A.13) and the result of the integration over w_1 that the relevant integral is

$$\int_{-\infty}^{\infty} w^\alpha (1 - w)^\beta {}_2F_1(a, b, c, w) dw \quad (\text{A.14})$$

As in the case of the integration over w_1 we want to use the identity (see **7.152.5** in [35])

$$\int_0^1 w^\alpha (1 - w)^\beta {}_2F_1(a, b, c, w) dw = \frac{\Gamma(\alpha + 1) \Gamma(\beta + 1)}{\Gamma(\alpha + \beta + 2)} {}_3F_2(a, b, \alpha + 1; c, \alpha + \beta + 2; 1) \quad (\text{A.15})$$

and thus split the integration in Eq. (A.14) as in Eq. (A.4). The integral $(1, \infty)$ is obtained from Eq. (A.15) by substitution $w \rightarrow 1/w$ and with the help of the identity Eq. (A.10). The result reads

$$\begin{aligned}
& \int_1^{\infty} w^\alpha (1 - w)^\beta {}_2F_1(a, b, c, w) dw = \quad (\text{A.16}) \\
& (-1)^{a+\beta} \frac{\Gamma(a - \alpha - \beta - 1) \Gamma(\beta + 1)}{\Gamma(a - \alpha)} \frac{\Gamma(c) \Gamma(b - a)}{\Gamma(b) \Gamma(c - a)} {}_3F_2(a, a - c + 1, a - \alpha - \beta - 1; a - b + 1, a - \alpha; 1) + \\
& (-1)^{b+\beta} \frac{\Gamma(b - \alpha - \beta - 1) \Gamma(\beta + 1)}{\Gamma(b - \alpha)} \frac{\Gamma(c) \Gamma(a - b)}{\Gamma(a) \Gamma(c - b)} {}_3F_2(b, b - c + 1, b - \alpha - \beta - 1; -a + b + 1, b - \alpha; 1)
\end{aligned}$$

The sum of Eq. (A.15) and Eq. (A.16) gives the contribution from the integration $(0, \infty)$. As in Eq. (A.6) the missing part $(-\infty, 0)$ is obtained from integral $(0, 1)$ substituting $w \rightarrow 1/(1 - w)$ and using identities Eq. (A.9) and Eq. (A.10)

$$\begin{aligned}
& \int_{-\infty}^0 w^\alpha (1 - w)^\beta {}_2F_1(a, b, c, w) dw = \quad (\text{A.17}) \\
& (-1)^\alpha \frac{\Gamma(a - \alpha - \beta - 1) \Gamma(\alpha + 1)}{\Gamma(a - \beta)} \frac{\Gamma(c) \Gamma(b - c)}{\Gamma(b) \Gamma(c - a)} {}_3F_2(a, b - c, a - \alpha - \beta - 1; -a - b + 1, a - \beta; 1) + \\
& (-1)^\alpha \frac{\Gamma(b - \alpha - \beta - 1) \Gamma(\alpha + 1)}{\Gamma(b - \beta)} \frac{\Gamma(c) \Gamma(a - b)}{\Gamma(a) \Gamma(c - b)} {}_3F_2(-b + c, a, b - \alpha - \beta - 1; c, b - \beta; 1)
\end{aligned}$$

Finally summing the contributions of Eq. (A.15), Eq. (A.16) and Eq. (A.17) and plugging it into Eq. (A.14) we obtain the contribution to Eq. (A.1) coming from the integration over the holomorphic variables z_1 and z_2 . The corresponding expression reads

$$\begin{aligned}
& - (-1)^{\Delta_{\tilde{0}1} - \Delta_{12}} B[1 + \gamma_2 - \Delta_{12}, 1 - \Delta_{2\tilde{0}}] B[1 + \Delta_{12}, \Delta_{\tilde{0}1} - \Delta_{12}] \quad (\text{A.18}) \\
& \times {}_3F_2[\{\gamma_1, -\Delta_{\tilde{0}1}, 1 + \gamma_2 - \Delta_{12}\}, \{-\Delta_{12}, 2 + \gamma_2 - \Delta_{12} - \Delta_{2\tilde{0}}\}, 1]
\end{aligned}$$

$$\begin{aligned}
& - (-1)^{\Delta_{\bar{0}1} - \Delta_{12}} B[1 + \gamma_2 - \Delta_{12}, 1 - \Delta_{2\bar{0}}] B(1 + \Delta_{12}, \Delta_{\bar{0}1} - \Delta_{12}) \\
& \times {}_3F_2(\{\gamma_1, -\Delta_{\bar{0}1}, 1 + \gamma_2 - \Delta_{12}\}, \{-\Delta_{12}, 2 + \gamma_2 - \Delta_{12} - \Delta_{2\bar{0}}\}, 1] \\
& - (-1)^{-\Delta_{\bar{0}1} + \Delta_{12}} B(1 + \gamma_2 - \Delta_{12}, 1 - \Delta_{2\bar{0}}] B(1 + \Delta_{12}, \Delta_{\bar{0}1} - \Delta_{12}) \\
& \times {}_3F_2(\{\gamma_1, -\Delta_{\bar{0}1}, 1 + \gamma_2 - \Delta_{12}\}, \{-\Delta_{12}, 2 + \gamma_2 - \Delta_{12} - \Delta_{2\bar{0}}\}, 1] \\
& - (-1)^{\gamma_2 - \Delta_{\bar{0}1}} B(1 + \gamma_2 - \Delta_{12}, -1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}] B(1 + \Delta_{12}, \Delta_{\bar{0}1} - \Delta_{12}) \\
& \times {}_3F_2(\{\gamma_1, -\Delta_{\bar{0}1}, 1 + \gamma_2 - \Delta_{12}\}, \{-\Delta_{12}, 2 + \gamma_2 - \Delta_{12} - \Delta_{2\bar{0}}\}, 1] \\
& - (-1)^{\gamma_2 + \Delta_{\bar{0}1} - 2\Delta_{12}} B(1 - \gamma_1 + \gamma_2 - \Delta_{12}, -1 + \gamma_1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}] B(1 + \gamma_1 + \Delta_{12}, \Delta_{\bar{0}1} - \Delta_{12}) \\
& \times {}_3F_2(\{\gamma_1, 1 + \gamma_1 + \Delta_{12}, -1 + \gamma_1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}\}, \{1 + \gamma_1 + \Delta_{\bar{0}1}, \gamma_1 - \gamma_2 + \Delta_{12}\}, 1] \\
& - \frac{(-1)^{\gamma_1 - \Delta_{\bar{0}1} - 2(\gamma_2 - \Delta_{12}) - \Delta_{12} - \Delta_{2\bar{0}}} B(-\gamma_1 - \Delta_{\bar{0}1}, 1 + \gamma_1 + \Delta_{12}) B(1 + \Delta_{12}, -\Delta_{12}) B(-1 + \gamma_1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}, 1 - \Delta_{2\bar{0}}]}{B(1 - \gamma_1, \gamma_1)} \\
& \times {}_3F_2(\{\gamma_1, 1 + \gamma_1 + \Delta_{12}, -1 + \gamma_1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}\}, \{1 + \gamma_1 + \Delta_{\bar{0}1}, \gamma_1 - \gamma_2 + \Delta_{12}\}, 1] \\
& - \frac{(-1)^{\gamma_1 - \Delta_{\bar{0}1} - 2(\gamma_2 - \Delta_{12}) + \Delta_{12} - \Delta_{2\bar{0}}} B(-\gamma_1 - \Delta_{\bar{0}1}, \Delta_{\bar{0}1} - \Delta_{12}) B(1 + \Delta_{12}, -\Delta_{12}) B(-1 + \gamma_1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}, 1 - \Delta_{2\bar{0}}]}{B(1 + \Delta_{\bar{0}1}, -\Delta_{\bar{0}1})} \\
& \times {}_3F_2(\{\gamma_1, 1 + \gamma_1 + \Delta_{12}, -1 + \gamma_1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}\}, \{1 + \gamma_1 + \Delta_{\bar{0}1}, \gamma_1 - \gamma_2 + \Delta_{12}\}, 1] \\
& - (-1)^{-\gamma_1 + \Delta_{\bar{0}1} - \Delta_{12} - \Delta_{2\bar{0}}} B(1 + \gamma_1 + \Delta_{12}, \Delta_{\bar{0}1} - \Delta_{12}) B(-1 + \gamma_1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}, 1 - \Delta_{2\bar{0}}] \\
& \times {}_3F_2(\{\gamma_1, 1 + \gamma_1 + \Delta_{12}, -1 + \gamma_1 - \gamma_2 + \Delta_{12} + \Delta_{2\bar{0}}\}, \{1 + \gamma_1 + \Delta_{\bar{0}1}, \gamma_1 - \gamma_2 + \Delta_{12}\}, 1] \\
& - (-1)^{\gamma_2 + \Delta_{\bar{0}1} - 2\Delta_{12}} B(\Delta_{\bar{0}1} - \Delta_{12}, 2 + \gamma_2] B(-1 + \gamma_1 - \gamma_2 + \Delta_{12}, 1 + \gamma_2 - \Delta_{12}) \\
& \times {}_3F_2(\{2 + \gamma_2, 1 + \gamma_2 - \Delta_{12}, \Delta_{2\bar{0}}\}, \{2 - \gamma_1 + \gamma_2 - \Delta_{12}, 2 + \gamma_2 + \Delta_{\bar{0}1} - \Delta_{12}\}, 1] \\
& + \frac{(-1)^{-\Delta_{\bar{0}1}} B(2 + \gamma_2, 1 - \Delta_{2\bar{0}}] B(1 + \Delta_{12}, -\Delta_{12})}{(1 + \gamma_1 + \Delta_{12}) B(\gamma_1, 2 + \Delta_{12})} \\
& \times {}_3F_2(\{2 + \gamma_2, 1 + \gamma_1 + \Delta_{12}, 1 - \Delta_{\bar{0}1} + \Delta_{12}\}, \{2 + \Delta_{12}, 3 + \gamma_2 - \Delta_{2\bar{0}}\}, 1] \\
& - \frac{(-1)^{\gamma_2 - \Delta_{\bar{0}1}} B(2 + \gamma_2, -2 - \gamma_2 + \Delta_{2\bar{0}}] B(1 + \Delta_{12}, -\Delta_{12})}{(1 + \gamma_1 + \Delta_{12}) B(\gamma_1, 2 + \Delta_{12})} \\
& \times {}_3F_2(\{2 + \gamma_2, 1 + \gamma_1 + \Delta_{12}, 1 - \Delta_{\bar{0}1} + \Delta_{12}\}, \{2 + \Delta_{12}, 3 + \gamma_2 - \Delta_{2\bar{0}}\}, 1] \\
& + (-1)^{\Delta_{\bar{0}1}} B(2 + \gamma_2, 1 - \Delta_{2\bar{0}}] B(1 + \gamma_1 + \Delta_{12}, -1 - \Delta_{12}) \\
& \times {}_3F_2(\{2 + \gamma_2, 1 + \gamma_1 + \Delta_{12}, 1 - \Delta_{\bar{0}1} + \Delta_{12}\}, \{2 + \Delta_{12}, 3 + \gamma_2 - \Delta_{2\bar{0}}\}, 1] \\
& - \frac{(-1)^{-2\Delta_{\bar{0}1} - 2(\gamma_2 - \Delta_{12}) - \Delta_{12} - \Delta_{2\bar{0}}} B(1 + \Delta_{12}, -\Delta_{12}) B(-1 - \gamma_2 - \Delta_{\bar{0}1} + \Delta_{12} + \Delta_{2\bar{0}}, 1 - \Delta_{2\bar{0}}]}{(\gamma_1 + \Delta_{\bar{0}1}) B(\gamma_1, 1 + \Delta_{\bar{0}1})} \\
& \times {}_3F_2(\{-\Delta_{\bar{0}1}, 1 - \Delta_{\bar{0}1} + \Delta_{12}, -1 - \gamma_2 - \Delta_{\bar{0}1} + \Delta_{12} + \Delta_{2\bar{0}}\}, \{1 - \gamma_1 - \Delta_{\bar{0}1}, -\gamma_2 - \Delta_{\bar{0}1} + \Delta_{12}\}, 1]
\end{aligned}$$

$$\begin{aligned}
& + \frac{(-1)^{-2\Delta_{\tilde{0}1}-2(\gamma_2-\Delta_{12})+\Delta_{12}-\Delta_{2\tilde{0}}} B(1+\Delta_{12}, -\Delta_{12}] B(-1-\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}+\Delta_{2\tilde{0}}, 1-\Delta_{2\tilde{0}}]}{(\gamma_1+\Delta_{\tilde{0}1})B(\gamma_1, 1+\Delta_{\tilde{0}1}]} \\
& \times {}_3F_2(\{-\Delta_{\tilde{0}1}, 1-\Delta_{\tilde{0}1}+\Delta_{12}, -1-\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}+\Delta_{2\tilde{0}}\}, \{1-\gamma_1-\Delta_{\tilde{0}1}, -\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}\}, 1] \\
& - (-1)^{\gamma_2-\Delta_{12}} B(\Delta_{\tilde{0}1}-\Delta_{12}, -1-\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}+\Delta_{2\tilde{0}}] B(-1+\gamma_1-\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}, 1+\gamma_2-\Delta_{12}-\Delta_{2\tilde{0}}] \\
& \times {}_3F_2(\{\Delta_{2\tilde{0}}, -1+\gamma_1-\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}, -1-\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}+\Delta_{2\tilde{0}}\}, \{-1-\gamma_2+\Delta_{2\tilde{0}}, -\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}\}, 1] \\
& - \frac{(-1)^{\gamma_2-\Delta_{\tilde{0}1}-\Delta_{12}} B(1+\Delta_{12}, -\Delta_{12}] B(-1+\gamma_1-\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}, 1+\gamma_2-\Delta_{12}-\Delta_{2\tilde{0}}]}{(-1-\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}+\Delta_{2\tilde{0}})B(1-\Delta_{\tilde{0}1}+\Delta_{12}, -1-\gamma_2+\Delta_{2\tilde{0}}]} \\
& \times {}_3F_2(\{\Delta_{2\tilde{0}}, -1+\gamma_1-\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}, -1-\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}+\Delta_{2\tilde{0}}\}, \{-1-\gamma_2+\Delta_{2\tilde{0}}, -\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}\}, 1] \\
& - \frac{(-1)^{\gamma_2-\Delta_{\tilde{0}1}} B(1+\Delta_{12}, -\Delta_{12}] B(-1-\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}+\Delta_{2\tilde{0}}, 2+\gamma_2-\Delta_{2\tilde{0}}]}{(-1+\gamma_1-\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}})B(\gamma_1, -\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}]} \\
& \times {}_3F_2(\{\Delta_{2\tilde{0}}, -1+\gamma_1-\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}, -1-\gamma_2-\Delta_{\tilde{0}1}+\Delta_{12}+\Delta_{2\tilde{0}}\}, \{-1-\gamma_2+\Delta_{2\tilde{0}}, -\gamma_2+\Delta_{12}+\Delta_{2\tilde{0}}\}, 1]
\end{aligned}$$

The integration over antiholomorphic variables \bar{z}_1 and \bar{z}_2 now can be easily performed using this last result.

References

- [1] J. Bartels, Z. Phys. C **60** (1993) 471.
- [2] J. Bartels and M. Wüsthoff, Z. Phys. C **66** (1995) 157.
- [3] M. Braun and G. P. Vacca, Eur. Phys. J. C **4**, (1998) 85.
- [4] A. Schwimmer, Nucl. Phys. B **94**, (1975) 445.
- [5] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. **100**, (1983) 1.
- [6] L. N. Lipatov, Sov. J. Nucl. Phys. **23** (1976) 338 [Yad. Fiz. **23** (1976) 642];
E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45** (1977) 199 [Zh. Eksp. Teor. Fiz. **72** (1977) 377];
I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822 [Yad. Fiz. **28** (1978) 1597].
- [7] L. N. Lipatov, Phys. Rept. **286** (1997) 131.
- [8] V. S. Fadin and L. N. Lipatov, Phys. Lett. B **429** (1998) 127;
M. Ciafaloni and G. Camici, Phys. Lett. B **430** (1998) 349;
V. S. Fadin and R. Fiore, Phys. Lett. B **610** (2005) 61 [Erratum-ibid. B **621** (2005) 61];
V. S. Fadin and R. Fiore, Phys. Rev. D **72** (2005) 014018.
- [9] I.I.Balitsky, Nucl. Phys. **B463** (1996) 99.
- [10] Yu.V.Kovchegov, Phys. Rev. **D60** (1999) 034008; **D61** (2000) 074018.
- [11] A. H. Mueller, Nucl. Phys. B **415** (1994) 373.
- [12] E. Levin and K. Tuchin, Nucl. Phys. B **573** (2000) 833;
M. Braun, Eur. Phys. J. C **16** (2000) 337;
H. Weigert, Nucl. Phys. A **703** (2002) 823;
N. Armesto and M. A. Braun, Eur. Phys. J. C **20** (2001) 517;
K. Golec-Biernat, L. Motyka and A. M. Staśto, Phys. Rev. D **65** (2002) 074037;
G. Chachamis, M. Lublinsky and A. Sabio Vera, Nucl. Phys. A **748** (2005) 649.
- [13] K. Golec-Biernat and A. M. Staśto, Nucl. Phys. B **668** (2003) 345.
- [14] K. Rummukainen and H. Weigert, Nucl. Phys. A **739** (2004) 183.
- [15] E. Levin and K. Tuchin, Nucl. Phys. A **691** (2001) 779;
J. Kwieciński and A. M. Staśto, Phys. Rev. D **66** (2002) 014013;
E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. A **708** (2002) 327;
S. Bondarenko, M. Kozlov and E. Levin, Nucl. Phys. A **727** (2003) 139 .
- [16] A. H. Mueller and D. N. Triantafyllopoulos, Nucl. Phys. B **640** (2002) 331;
D. N. Triantafyllopoulos, Nucl. Phys. B **648** (2003) 293;
L. Motyka, Phys. Lett. B , arXiv:hep-ph/0509270.

- [17] J. Bartels and E. Levin, Nucl. Phys. B **387** (1992) 617.
- [18] A. M. Staśto, K. Golec-Biernat and J. Kwieciński, Phys. Rev. Lett. **86** (2001) 596.
- [19] S. Munier and R. Peschanski, Phys. Rev. Lett. **91** (2003) 232001;
S. Munier and R. Peschanski, Phys. Rev. D **69**, 034008 (2004);
S. Munier and R. Peschanski, Phys. Rev. D **70**, 077503 (2004).
- [20] S. Bondarenko, E. Gotsman, E. Levin and U. Maor, Nucl. Phys. A **683** (2001) 649.
- [21] M. A. Braun, Phys. Lett. B **483** (2000) 115.
- [22] M. A. Braun, Eur. Phys. J. C **33** (2004) 113.
- [23] M. A. Braun, arXiv:hep-ph/0504002.
- [24] M. A. Braun, Phys. Lett. B **632** (2006) 297.
- [25] S. Bondarenko and L. Motyka, Phys. Rev. D **75**, (2007) 114015.
- [26] S. Bondarenko, Nucl. Phys. A **792** (2007) 264.
- [27] L.N.Lipatov, in "Perturbative QCD", ed. A.H.Mueller, World. Sci. Singapore (1989).
- [28] L.N.Lipatov, Sov. Phys. JETP **63** (1986) 904, Nucl. Phys. B **715** (1991) 641, Phys. Rept. **286** (1997) 131.
- [29] J. Bartels, L. N. Lipatov and G. P. Vacca, Nucl. Phys. B **706** (2005) 391.
- [30] R. Peschanski, Phys. Lett. B **409** (1997) 491.
- [31] G. P. Korchemsky, Nucl. Phys. B **550** (1999) 397.
- [32] Y. Hatta and A. H. Mueller, Nucl. Phys. A **789** (2007) 285.
- [33] E. Levin and K. Tuchin, Nucl. Phys. B **573** (2000) 833;
E. Levin and K. Tuchin, Nucl. Phys. A **691** (2001) 779.
- [34] Abramowitz, M., and Stegun, I. S. 1972, Handbook of Mathematical Functions (New York: Dover)
- [35] I. S. Gradshteyn and I. M. Ryzhik, in Tables of Integrals, Series and Products, edited by A. Jeffrey (Academic, New York, 1980)